Mathematical Modelling in Building Construction

One of the most common features in all cultures is the design and construction of buildings for a variety of purposes such as in houses, office buildings, warehouses, stores, specialty buildings such as hospitals, airports, fire, police and emergency services, religious and a multitude of other structures.

There will usually be, within each culture, buildings that are designed for a variety of reasons such as through history, climate, geography and so forth. They may be adorned in different manners due to such reasons as the ethnicity of the people, regional differences, language differences and more factors possibly too numerous to mention. The constant between all forms of construction, design and erection of buildings is that the Mathematics that is used in almost universal and rely only upon a fairly small number of strategies and elements.

The majority of buildings may comprise many different elements such as rooms, offices, baths and showers, additional rooms, decks and so forth but they will in most cases be made from a mixture of squares, rectangles, triangles and circles. As a consequence it is important that the features of each of these is well known so that the structure can be built to standards set down by the various building authorities.

When constructing a building it is essential that the correct sizes of materials are used when building so that there are not failures with the material that could lead to injury either to the person who is doing the building or to the owner/occupier of the dwelling when it is finished. This is the key component of any building project to ensure that safety is the paramount concern. Provided that the correct materials are used and installed properly, the main concern is ensuring that the building is constructed correctly and that all guidelines and standards are met with the building.

It is possible to construct any building to any size at all but there are certain restrictions that have to be followed and enforced. In general, a structure will be built upon land that is owned or legally acquired by the person who wants the building to be erected.

A person will usually want the structure to be built so that it meets the costs of a budget and so there will need to be a reduction in waste of materials and resources. Hence, it is important to recognise that materials for the structure will generally be obtained in sizes that are common and which have been used over a long period of time.

Building Measurement Units

The most common measurement in building construction is length. Today there is still a mixture of two units of length measurement and these include the historical units linked to the English empire and these are the Imperial Units of feet and inches (ft, in) and those of the modern era with Metric Units of metres and millimetres.

Even in the modern buildings, the majority of material suppliers still loosely base their measurements upon the Imperial system. This is why it is important to recognise that certain sizes are generally used and when these are known then it is much more likely that structures will use appropriate measures to avoid ineffiencies in construction.

Whether or not imperial or metric measure is used, it can be helpful to understand the basic units that were used regularly and those that are used in common now.

Imperial Measure
The common Imperial measure of length is the foot. Irrespective of it's origin, the unit of measure was called 1 foot. From this was the longer unit of the yard (where 1 yard was equal in length to 3 feet). There were smaller units than the foot and these were called inches (1 foot was comprised of twelve equal parts, each of which was called an inch). It was common to break an inch into equal smaller portions and these were called ½ an inch, ¼ of an inch, $^1/_8$ of an inch, $^1/_{16}$ of an inch, $^1/_{32}$ of an inch and $^1/_{64}$ of an inch. Because of the accuracy of measurements it was not considered feasible in any building construction to use any smaller units.

This means that the common units of measurement in the Imperial system were;

1 yard (1 yd)	=	3 feet (3ft or 3')
1 foot (1 ft or 1')	=	12 inches (12in. or 12")

Metric Measure
Many European countries and now generally worldwide the majority of countries internationally recognise the decimalised metric system of measurement. The standard unit of length measurement is the metre. The metre is subdivided into 100 smaller and equal units called the centimetre (where centi means "$^1/_{100}$ of"). Once again the centimetre is divided into 10 equal parts called millimetres (where milli means "$^1/_{1000}$ of"). The connection between these metric units is;

1 metre (1m)	=	100 centimetres (100cm)
1 centimetre (1cm)	=	10 millimetres (10mm)
1 metre (1m)		1000 millimetres (1000mm)

Connection between Imperial and Metric Units
The conversion factor that is used for measurement is that 1 yard is equivalent to 0.914m. This means that 0.914m is equivalent to 914mm (or 14mm over 900mm). In the past, the yard was a common unit of length measurement and so now its replacement is 900mm. This is the reason why the figure of 900mm is regularly applied in building construction.

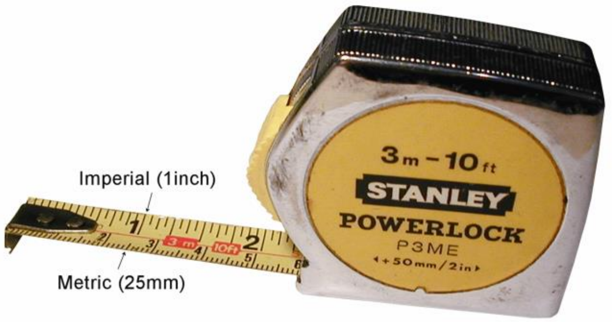
Since one yard was equal to 3 feet, this means that 300mm is often used in buildings because it is $^1/_3$ of 900mm.

Even though the following are not entirely accurate as far as conversion between Imperial and Metric measures are concerned, they are extremely close;

1 foot (1')	=	30 centimetres (30cm)
1 foot (1')	=	300 millimetres (300mm)
1 inch (1in. or 1")	=	2.5 centimetres (2.5cm)
1 inch (1in. or 1")	=	25 millimetres (25mm)

Length and Area Measurement

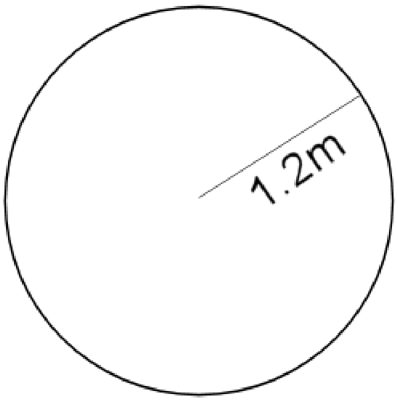
For measuring in a linear fashion a tape measure will generally be used and quite often the tape will include both metric and imperial measures as shown below;



In most instances, lengths will be along straight edges with shapes such as rectangles, squares and triangles being used but with construction people may have other shapes with irregular sides.

The other common shape used in construction will usually be the circle, semi-circle or sector. In these instances it may be necessary to calculate linear measurement around a curved side which will be the circumference. It is usually possible to determine, using a tape measure the length of the radius or diameter and then to use the circumference formula to work out the linear curved length. Examples are given below (answers rounded up to next whole centimetre);

Circumference of circle

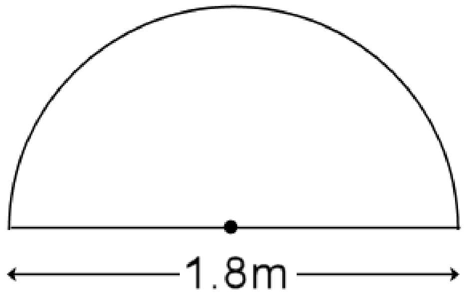


$$\begin{aligned} C &= 2\pi r \\ &= 2 \times \pi \times 1.2 \\ &= 7.539822369 \\ &\approx 7.54 \end{aligned}$$

Alternatively, the diameter (2.4m) could be used as shown;

$$\begin{aligned} C &= \pi d \\ &= \pi \times 2.4 \\ &= 7.539822369 \\ &\approx 7.54 \end{aligned}$$

Circumference of semi-circle



Find the Perimeter of the semi-circle.

$$\begin{aligned} C &= \pi d \\ &= \pi \times 1.8 \\ &= 5.654866776 \\ &\approx 5.66 \end{aligned}$$

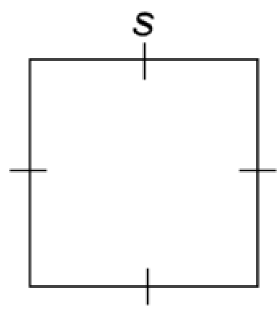
Perimeter is curved length plus straight length giving;

$$\begin{aligned} P &= 5.66 + 1.8 \\ &= 7.46 \end{aligned}$$

Due to building, round up for practical reasons.

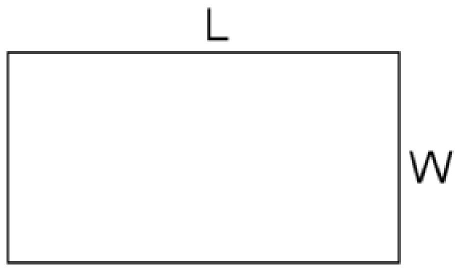
Frequently Used Area Formulae in Construction

Square
Each side equal in length (call it ***s*** units)



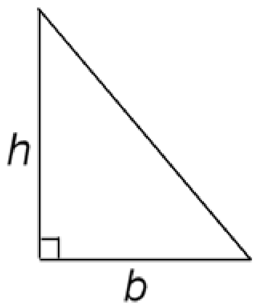
$$A = s^2$$

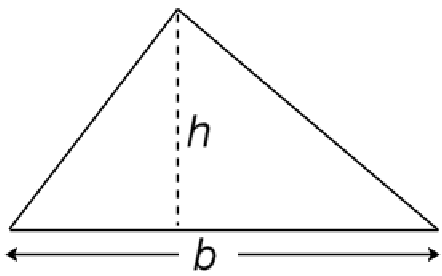
Rectangle
Longest side or length (call it ***L*** units) and shorter side or width (call it ***w*** units)



$$A = LW$$

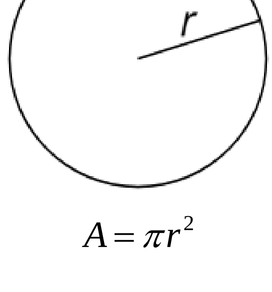
Right Triangle or non-right Triangle
From one side called the base (of length ***b*** units), the perpendicular height to the third vertex of the triangle is called the height (written as ***h*** units)





$$A = \frac{bh}{2}$$

Circle
If diameter (***d***) given then radius (***r***) will simply be half the value.

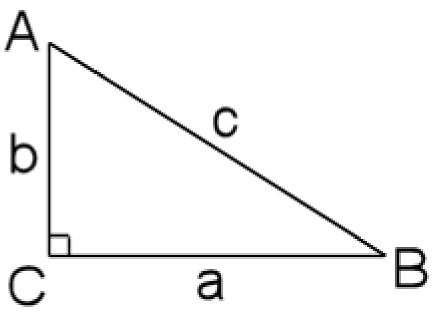


$$A = \pi r^2$$

Pythagoras' Theorem

Not all building or construction uses right angles but this is in most cases accepted as the standard. A knowledge of Pythagoras' Theorem is extremely useful to ensure that elements such as corners and placement of objects are done so that they are in general perpendicular or at right angles to each other.

In a right triangle, the right angle is always opposite the hypotenuse and the square on the hypotenuse is equal to the sum of the squares on the other two sides. In the right angled triangle shown below, *c* is the hypotenuse and the two shorter sides are *a* and *b* and the right angle is $\angle$ACB.



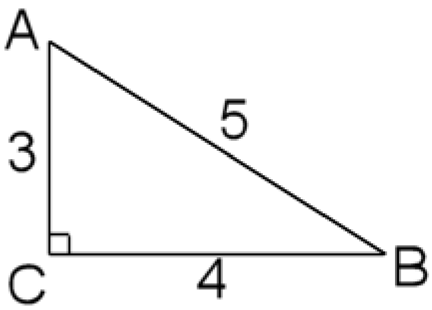
For the triangle above, the application of Pythagoras' Theorem (in terms of the side lengths a, b, c or AB, BC, CA) is;

Pythagoras' Theorem; $c^2 = a^2 + b^2$

$$AB^2 = AC^2 + CB^2$$

Pythagorean Triads

A basic right triangle has two sides of length 3 units and 4 units with the third side (the Hypotenuse) of length 5 units. This is called the 3,4,5 triad which is a right triangle and this is shown below;



$AB = 5$ $AC = 3$ $CB = 4$

Using Pythagoras' Theorem

$$c^2 = a^2 + b^2$$

$$5^2 = 4^2 + 3^2$$

$$25 = 16 + 9$$

$$25 = 25$$

This is TRUE and hence the triangle is right angled.

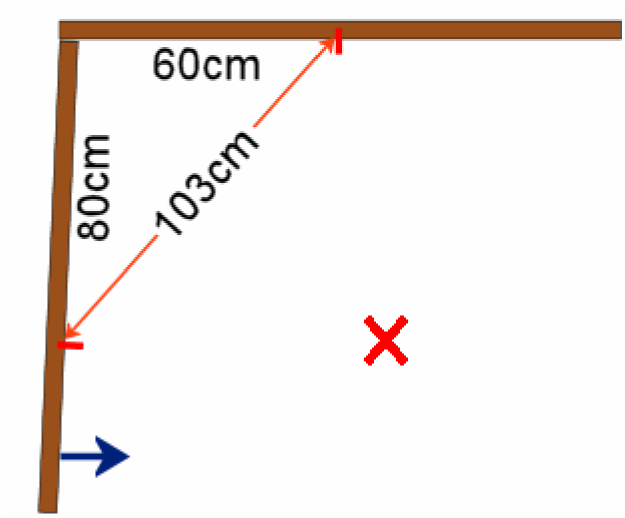
Using Pythagorean Triads in Building and Construction
There are many Pythagorean Triads but the 3,4,5 triangle is generally used because of its simplicity. The units can be chosen and do not necessarily have to be restricted to the basic units of length measure such as metres, centimetres, feet and so forth. In fact any length unit can be used as the basis and so a right triangle can be any multiple of the Pythagorean Triad.

This means that if the unit of measurement chosen was (say) 15cm, then the size of the Pythagorean Triad, based on 3,4,5 would be equal to 45,60,75 (3x15, 4x15, 5x15). The unit used may always be the same or indeed it may change constantly.

To check that a corner is at right angles (90°) it is simply a matter of measuring along two sides (shorter) then checking that the third side (hypotenuse) has the measurement given for the triad.

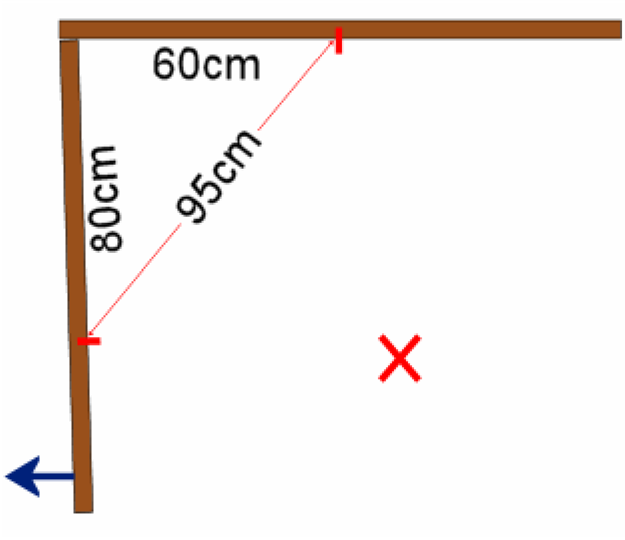
Consider the following two scenarios where it is important to have corners in a structure that are at right angles to each other. From the corner, measurements of 60cm (3x20cm) and 80cm (4x20cm) are measured and marked. Whether or not the corner is 'square' (meaning at right angles) will depend upon the third measurement which should be 100cm (5x20cm). In construction, before final fixing is done, there is provision for movement of components so that they align and are accurate. The diagrams below show situations where wall framing either needs to move inwards or outwards as well as the scenario where the wall frames are in the correct positions relative to each other.

a)



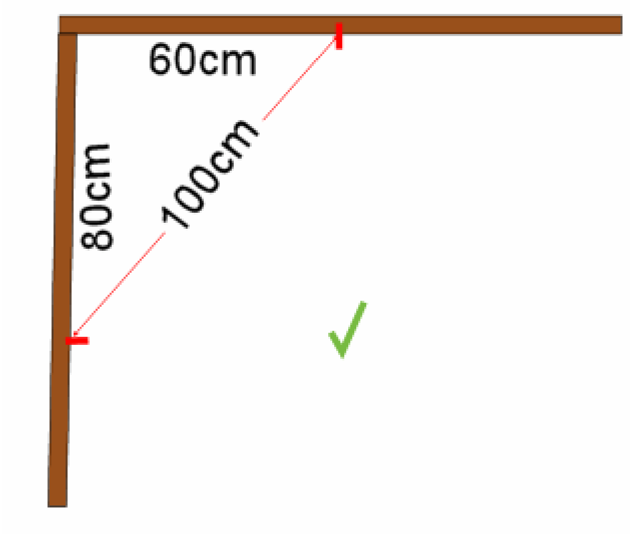
To make a right angle (or square corner in construction) it would be necessary to close the gap by moving the wall frame inwards until the length between marks was 100cm.

b)



To make the corner square in this instance it would be necessary to move the wall frame outwards until the correct distance of 100cm could be obtained.

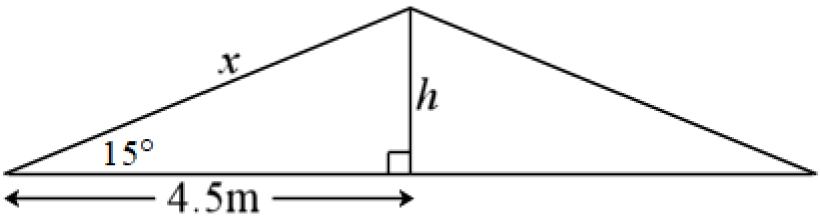
c)



In this case, the two wall frames would be perpendicular to each other.
Of course using ‘Real Life’ Mathematics it is always wise to check to ensure accuracy.
There are also builder’s squares that can be used also to aid in ensuring accuracy.

Using Trigonometry in Construction
Trigonometry literally means triangle measurement. In general, the frames that are used to support roofing material are triangular in shape. There are several reasons for having roofs made in a triangular shape and would most likely include the fact that triangles are a stable and rigid structure, materials used will be less and hence the cost will be less and also a sloping surface for a roof (its pitch) will be essential for being able to direct rain and other debris from remaining on the roof and causing possible structural problems.

Suppose a roof needs to be built so that it has a pitch of 15° . From the middle of the structure the distance to the edge of the building is 4.5m. If a roof frame is to be made in the shape of a triangle, what would be the height of the roof and what would be the length of the rafter.



Before trying to attempting to solve the problem there are three basic ratios used with triangles and these include the sine ratio, cosine ratio and tangent ratio and they all apply to a right angled triangle. The longest side length in a right angled triangle is always opposite the right angle and is always called the hypotenuse. From one of the angles (NOT the hypotenuse) in a right triangle it is possible to name the other two sides relative to the chosen angle. These two side lengths will be called the opposite and the adjacent.

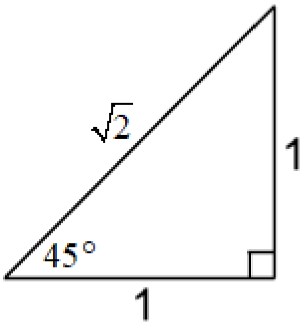
For a given angle, there will be three ratios, called;

Sine Ratio (abbreviated to sin)	Opposite : Hypotenuse
Cosine Ratio (abbreviated to cos)	Adjacent : Hypotenuse
Tangent Ratio (abbreviated to tan)	Opposite : Adjacent

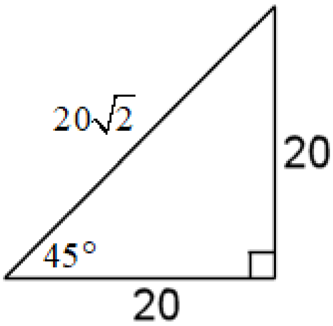
A ratio is a comparison of two values of equal type and so the ratios can be simplified. For instance a ratio of 90:180 can be written as 45:90 or 9:18 or 1:2 (in its simplest form). It is also possible to write a ratio as a rational number such as 1/2 or even as a decimal 0.5.

This means that a ratio will also imply that it can be written in decimal form through the process of division. If the idea is well understood, then the trigonometry and its usage is quite a simple process. This can be seen in the following situation;

The trigonometric ratios are to be determined for an angle of 45°. Such an angle is made when there are two equal length sides formed around the right angle. For convenience we will call the side lengths one unit.



Using Pythagoras' Theorem, the length of the hypotenuse can be found as $\sqrt{2}$ units. Any multiple of this triangle will also be a 45° triangle. This is shown below where all measurements are 20 times larger.



In the smaller triangle, the tangent ratio is;

$$\tan 45^\circ = \frac{1}{1} \\ = 1$$

While in the larger triangle the tangent ratio is;

$$\tan 45^\circ = \frac{20}{20} \\ = 1$$

Which means that irrespective of the physical size of the right triangle, the trigonometric ratio for a given angle will have the same value. Using the sine ratio in the 45° triangle;

$$\sin 45^\circ = \frac{1}{\sqrt{2}} \\ = 0.7071067812$$

The result is found from a scientific calculator as $1 \div \sqrt{2}$ or using the fraction keys $\frac{1}{\sqrt{2}}$.

Rather than having to construct a triangle, find the lengths of sides and determine the values of the ratios all the time, it is much easier to use a Scientific Calculator with measurement set to DEG(rees) and use the trigonometric function keys of sin, cos and tan.

Using the previous example, the value for $\sin 45°$ is found simply by pressing sin 4 5 = and the result of 0.7071067812 should appear in the display if found correctly.

Using Equations in Building Construction
An equation involves the calculation of a value in an equality expression where one of the variables is unknown. The method of solving an equation is to methodically re-organise by performing inverse operations until the value to be found is determined. Inverse operations include addition and subtraction as well as multiplication and division.

The most basic equations can be found from simple number facts such as;

5 + 9 = 14	Implies	5 = 14 – 9
	Or	9 = 14 – 5
9 x 6 = 54	Implies	6 = 54 ÷ 9
	Or	9 = 54 ÷ 6
19 – 7 = 12	Implies	19 = 7 + 12

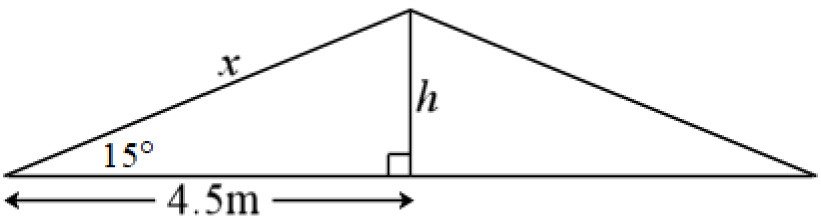
and so forth.

More advanced, yet still quite basic equations and there method of solution would be similar to the following;

1. $a+5=13$
   $a=13-5$
   $a=8$

2. $\frac{m}{7}=3$
   $m=3\times 7$
   $m=21$

Now by using these skills (trigonometry and equation solving) it is now possible to determine values without the need to actually go through a complicated process of physically performing measurements. This is shown with the example;



Where h and x have to be found as shown in the following;

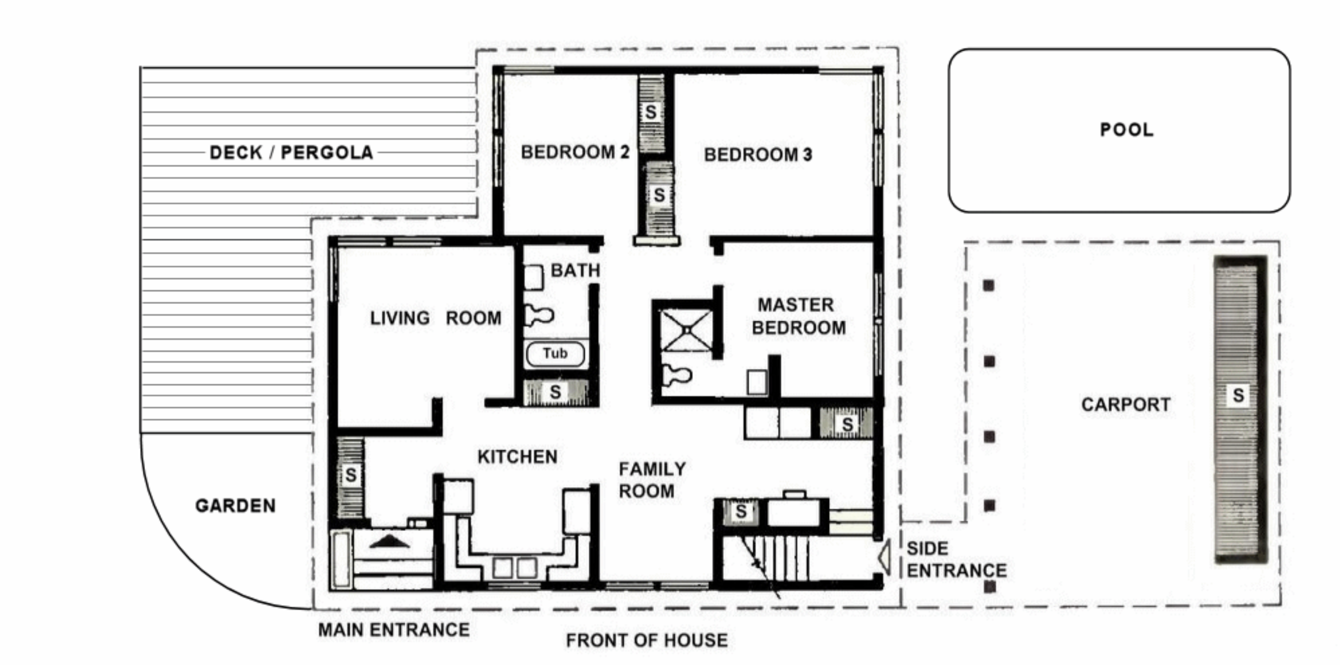
$$\tan 15° = \frac{h}{4.5}$$
$$h = 4.5 \times \tan 15°$$
$$h = 1.205771366$$
$$h \approx 1.206$$

Since the value of h refers to the height of a roof truss in building it would be most suitable to 'round off' to a suitable degree of accuracy and so the value is approximately 1.206m or using equivalent measurement it could be 1,206mm.

$$\cos 15° = \frac{4.5}{x}$$
$$x \times \cos 15° = 4.5$$
$$x = \frac{4.5}{\cos 15°}$$
$$x = 4.658742812$$
$$x \approx 4.659$$

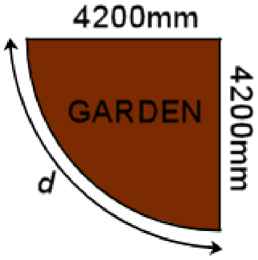
This means that the length of the rafter in the building would be approximately 4.659m or 4,659mm.

Now, looking at a house plan (similar to the one shown below)



It can be seen that length measurement forms a substantial component in any construction and it is important that accurate measurements are taken and suitable calculations of area are performed so that it is possible to determine the cost of materials needed. As can be seen, the majority of shapes are either square or rectangular but it is not unusual to have alternate shapes such as triangles or even parts of circles (such as the quadrant shaped garden).

Suppose the garden at the front of the house was in the shape of a quadrant (1/4 circle) and that it had straight edged sides made from timber of length 4200mm. What would be the distance around the curved edge that would be needed as another border. This could be determined using the following as a guide;



The distance to be found is one quarter the distance around the circle (circumference), this can be found using;

$$
\begin{aligned}
C &= \tfrac{1}{4} \times 2\pi r \\
&= \tfrac{1}{2} \times \pi r \\
&= \frac{\pi r}{2} \\
&= \frac{\pi \times 4.2}{2} \\
&= 6.597344573 \\
&= 6.597m \\
&= 6597mm
\end{aligned}
$$

Which means that the distance around the curved edge of the garden would be approximately 6,597mm.

In any construction, it is necessary to have exact measurement but when planning it is always best to have more material rather than less because it is usually possible to remove material but impossible or very difficult to add on to any building material. This would mean that in the garden shown above, it would be necessary to used 6,597mm of edging material but possibly best to allow for 6,600mm or 6.6m of edging.

It is impossible to cover every contingency in construction. It should be noted that the information given so far should illustrate how necessary it is to have a solid understanding of Mathematical skills for even the most basic of tasks in building and construction. There are many more skills required but those shown so far should highlight the need to have developed effective Mathematical knowledge.